Introduction to Circuits: Finding resistance

September 14, 2025

1 Background

1.1 Where are we now?

We developed a really strong suite of trick for solving problems in classical mechanics, and are now going to move on to something completely different, circuits! This part is slightly harder to motivate than our previous studies on classical mechanics, because circuits do not arise in nature. However, they are important for many practical engineering applications, and solving them provides an interesting test bed for a lot of problem solving techniques.

1.2 Finding resistances

We will start by focusing on the simplest case, finding the resistance of circuits. There are loads of important tricks for this, that I will just list here and discuss more in the session. These are listed roughly in order from easy to hard.

Feel free to skim these at first (read the node labeling one carefully) and then consult them as needed as you go through the problem sheet. Each technique will show up at least once in the handout.

1. **Series and parallel resistors**

$$R_{series} = R_1 + R_2 \tag{1}$$

$$R_{parallel} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \tag{2}$$

2. **Node labeling**: This is very useful. Consider the description given by Kalda (will go through in session).

> For complex circuits, it is easy to make mistakes while simplifying the circuit; typically, this happens when the remote nodes are connected with wires. To avoid mistakes, the following technique can be applied. Label all the resistors, e.g. with letters; if there is more than one battery, label the batteries, as well. Label also all the nodes, so that all the nodes connected with a plain wire bear the same label, and those which have no direct wire connection have different label. Then, start redrawing the circuit by marking (on a sheet of paper) one node and drawing all those resistors which are connected to it. Next, select another lead of one of the drawn resistors or batteries, mark the respective nodes and draw the resistors which are attached to that node; repeat the process until the entire circuit is redrawn.
>
> As an example, let us consider the last problem. We mark the nodes and resistors as shown in figure. Note that due to the wire connections, the node symbols appear in two different places.
>
> We start with drawing the node 'A', see the figure. Since the node 'A' is directly connected to the resistors '1', '2' and '3', we draw these resistors attached to the node 'A' as shown in figure. The other ends of the resistors '1' and '2' are fixed to the node 'C', hence we can connect the respective wires and designate the connection point by 'C'. Further, the other end of the resistor '3' is connected to the node 'B', so we draw a wire and mark its end with

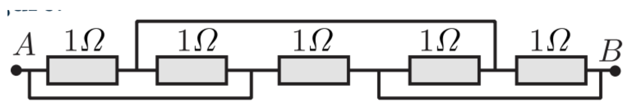

Figure 1: Initial example

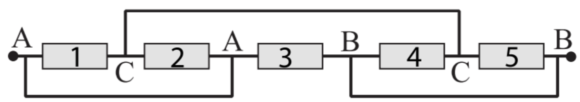

Figure 2: Redraw 1

'B'. Now, by noticing that the resistors '4' and '5' connect the nodes 'B' and 'C', it is easy to complete the circuit.

In the case of non-trivial circuit-redrawing tasks, it is highly recommended to use this technique of denoting resistors and nodes with letters and numbers (you don't want to make a mistake in redrawing!).

3. **Ammeters and voltmeter**: In the ideal case, ammeters can be considered 0 resistance, and voltmeters have infinite resistance. This means that ammeters can be considered as a short circuit (no voltage drop), and voltmeters as an open circuit (no current flow).
4. **Merging nodes**: If two nodes are at the same voltage, they can effectively be short circuited. If two nodes have zero current between them, they can be effectively be considered an open circuit. This kind of rewriting can be useful.
5. **Resistance by imaginary currents or voltages**: From Kalda:

   If the task is to find the resistance of a circuit between two leads, it is often useful to assume that either a voltage V is applied to the leads, or a current I is driven through these leads. Then we need to find the missing quantity (I or V , respectively), and calculate $R = V/I$.

2 Questions

2.1 Check your understanding

1. Determine the resistance between the leads of the circuit in figure 4.
2. Determine the reading of the ammeter in figure 5.

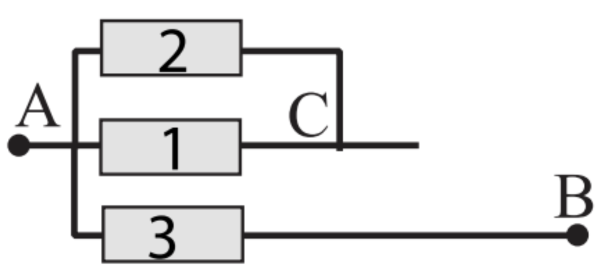

Figure 3: Redraw 2

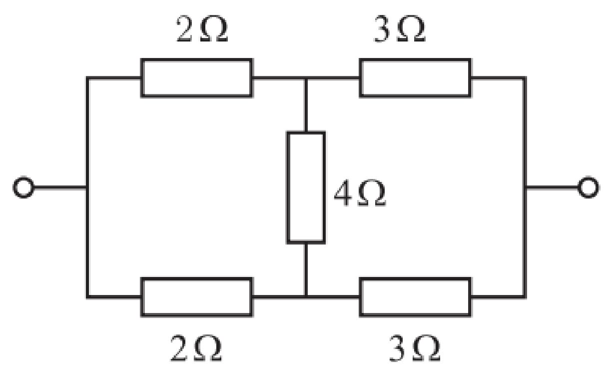

Figure 4:

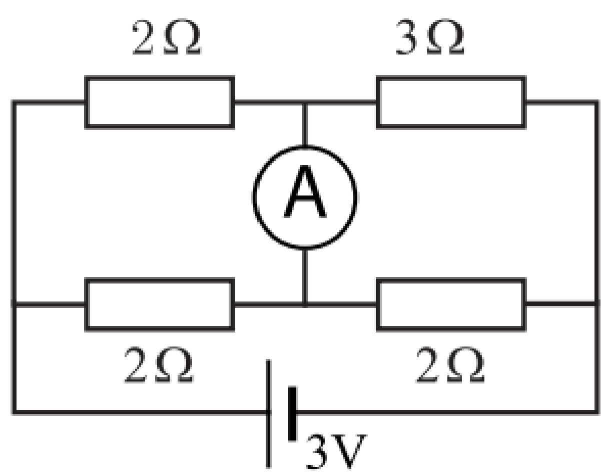

Figure 5:

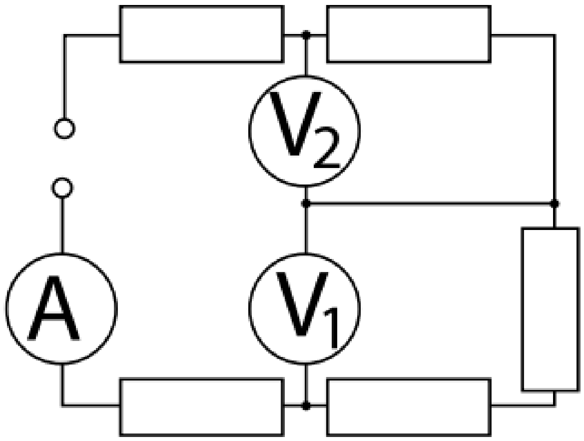

Figure 6:

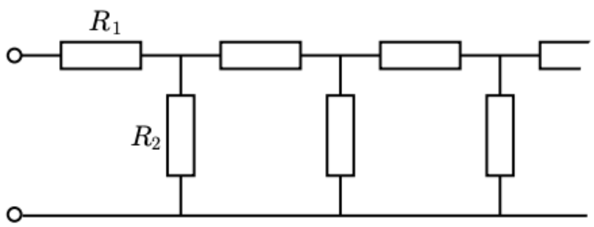

Figure 7:

3. The two voltmeters in the circuit in figure 6 are identical; their readings are $V_1 = 30$ V and $V_2 = 20$ V. The reading of the ammeter is I = 750 μA. All the five resistors have equal resistance R; find the numerical value of R.

4. Determine the resistance of the circuit in figure 7.

5. Find the resistance between the terminals A and B for the infinite chain shown in figure 8. The resistances are as shown and increase by a factor of two for each consecutive link.

6. Determine the resistance between opposing corners of a cube, the edges of which are made of wire, see figure 9; the resistance of one edge is 1 Ω

7. Determine the resistance between the output leads of the circuit in figure 10.

8. In figure 11, all the resistors have equal resistance $R = 1\Omega$ ammeters and the battery are ideal, $E = 1$ V. Determine the readings of all the ammeters.

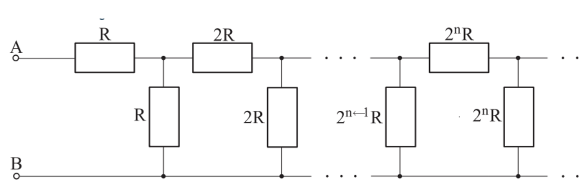

Figure 8:

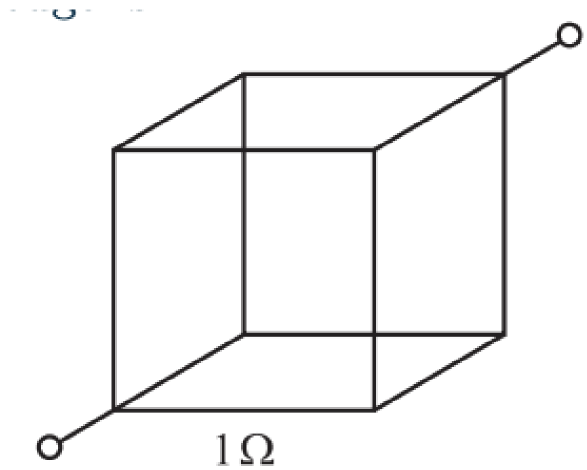

Figure 9:

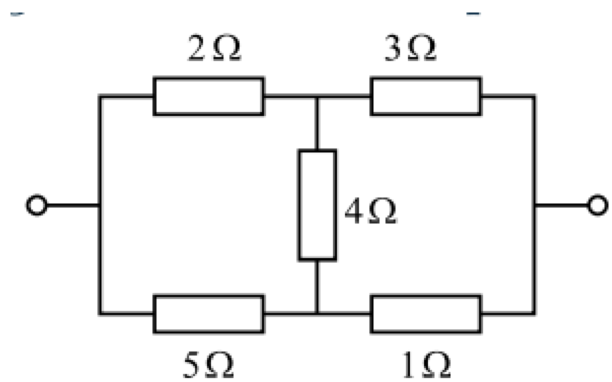

Figure 10:

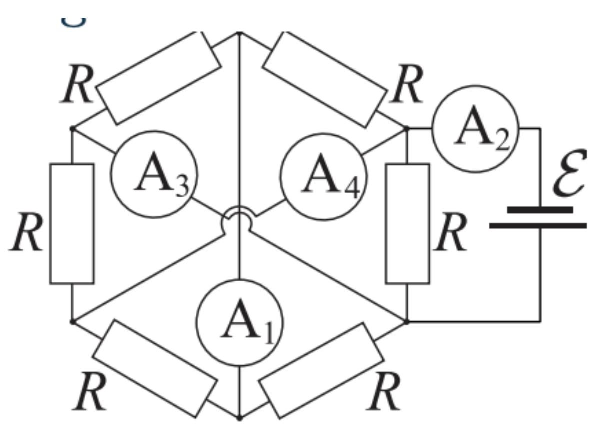

Figure 11:

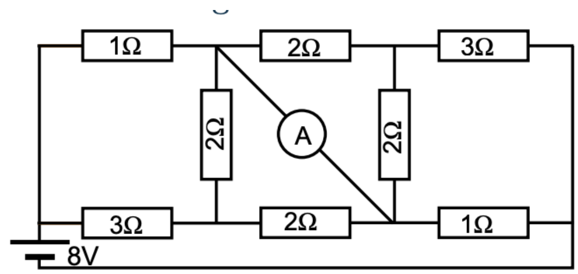

Figure 12:

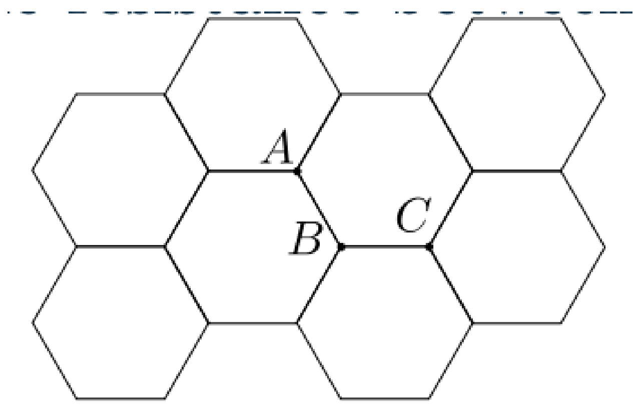

Figure 13:

9. Find the reading of the ammeter in the circuit in figure 12.
10. There is an infinite honeycomb lattice (see figure 13); the edges of the lattice are made of wire, and the resistance of each edge is R. Let us denote two neighbouring vertices of a vertex B by A and C. Determine the resistance between:
    (a) A and B
    (b) A and C